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Unsteady Stagnation-point Flow of a Second-grade Fluid

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Abstract *- The unsteady two-dimensional stagnation point flow of second-grade fluid impinging on an infinite plate is examined and solutions are obtained. It is assumed that the infinite plate at $y=0$ is oscillating with velocity $U\cos\Omega t$, the fluid occupies the entire upper half plane $y>0$ and it impinges obliquely on the plate. The governing partial differential equations are reduced to a system of ordinary differential equations by assuming a form of the streamfunction a priori. The resulting equations are, then, solved numerically using a shooting method for various values of the Weissenberg number, We. It is observed that the effect of the Weissenberg number is to decrease the velocity near the wall as it increases. Furthermore, analytical solutions are obtained for small and large values of frequency.*

Keywords: Unsteady, Stagnation-point, Oscillating plate, Non-Newtonian fluid.

Nomenclature

$A_1,\ A_2$	1st and 2nd Rivlin Ericksen tensors
$F(y), G(y,t)$	similarity variables
k	constant
p	fluid pressure
T	Cauchy Stress Tensor
V	velocity vector
u, v	velocity components along x and y axis
U	constant
We	Weissenberg number

Greek Symbols

α_1, α_2	viscoelastic parameters of the fluid
$\beta = \frac{\Omega}{k}$	frequency
γ	constant
$\varepsilon = \frac{U}{\sqrt{\nu k}}$	constant
η	non-dimensional varable
μ	fluid viscosity
ν	kinematic viscosity
τ_{12}	Shear stress component
$\tau = \Omega t$	non-dimensional variable
ψ	streamfunction
Ω	frequency

1. Introduction

In the past, the fluid flow near a stagnation point has been investigated extensively. Hiemenz [1] derived an exact solution of the steady flow of a Newtonian fluid impinging orthogonally on an infinite flat plate. Stuart [2], Tamada [3] and Dorrepaal [4] independently investigated the solutions of a stagnation point flow when the fluid impinges obliquely on the plate. Beard and Walters [5] used boundary-layer equations to study two-dimensional flow near a stagnation point of a viscoelastic fluid. Dorrepaal et al [6] investigated the behaviour of a viscoelastic fluid impinging on a flat rigid wall at an arbitrary angle of incidence. Labropulu et al [7] studied the oblique flow of a viscoelastic fluid impinging on a porous wall with suction or blowing.

Unsteady stagnation point flow of a Newtonian fluid has also been studied extensively. Rott [8] and Glauert [9] studied the stagnation point flow of a Newtonian fluid when the plate performs harmonic oscillations in its own plane. Srivastava [10]

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investigated the same problem for a non-Newtonian second grade fluid using the Karman-Pohlhausen method [11] to solve the resulting equations. Labropulu et al [12] used series methods to solve the unsteady stagnation point flow of a Walters' B' fluid impinging on an oscillating flat plate. Matunobu [13, 14] and Kawaguti and Hamano [15] examined the fundamental character of the unsteady flow near a stagnation point for a Newtonian fluid. Takemitsu and Matunobu [16] studied the oblique stagnation point flow for a Newtonian fluid and obtained the general features of a periodic stagnation point flow. The case when the stagnation point fluctuates along a solid boundary is especially interesting from the biomechanical point of view. This is because the wall shear stress experienced by blood vessels may be thought to be increased by pulsating blood flow near the mean position of fluctuating stagnation point [15, 17] and lead to vascular diseases [18].

In this work, the unsteady stagnation point flow of a viscoelastic second-grade fluid is examined and solutions are obtained. We assume that the infinite plate at $y = 0$ is oscillating with velocity $U\cos\Omega t$, the fluid occupies the entire upper half plane $y > 0$ and the fluid impinges obliquely on the plate. The governing partial differential equations are reduced to a system of ordinary differential equations by assuming a form of the streamfunction a priori. The resulting equations are, then, solved numerically using a shooting method for various values of the Weissenberg number, We. It is observed that the effect of the Weissenberg number is to decrease the velocity near the wall as it increases. Furthermore, analytical solutions are obtained for small and large values of frequency.

2. Flow Equations

The flow of a viscous incompressible non-Newtonian second-grade fluid, neglecting thermal effects and body forces, is governed by

$$div\, V = 0 \tag{1}$$
$$\rho\dot{V} = div\, T \tag{2}$$

when the constitutive equation for the Cauchy stress tensor T which describes second-grade fluid given by Rivlin and Ericksen [19] is

$$\left.\begin{aligned} T &= -pI + \mu A_1 + \alpha_1 A_2 + \alpha_2 A_1^2 \\ A_1 &= (grad\, V) + (grad\, V)^T \\ A_2 &= \dot{A}_1 + (grad\, V)^T A_1 + A_1 (grad\, V) \end{aligned}\right\} \tag{3}$$

Here V is the velocity vector field, p the fluid pressure function, ρ the constant fluid density, μ the constant coefficient of viscosity and α_1, α_2 the normal stress moduli. Dunn and Fosdick [20] and Dunn and Rajagopal [21] have shown that if the second-grade fluid described by (3) is to undergo motions which are compatible with Clausius-Duhem inequality [22] and the assumption that the free energy density of the fluid be locally at rest, then the material constants must satisfy the following restrictions:

$$\mu \geq 0, \qquad \alpha_1 \geq 0, \qquad \alpha_1 + a_2 = 0 \tag{4}$$

Considering the flow to be plane, we take $V = (u(x,y,t), v(x,y,t))$ and $p = p(x,y,t)$ so that the flow equations (1) to (3) take the form

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{5}$$

$$\begin{aligned} &\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \frac{1}{\rho}\frac{\partial p}{\partial x} = \nu\nabla^2 u + \frac{\alpha_1}{\rho}\Bigg\{\frac{\partial}{\partial t}\left(\nabla^2 u\right) \\ &+\frac{\partial}{\partial x}\left[2u\frac{\partial^2 u}{\partial x^2} + 2v\frac{\partial^2 u}{\partial x\partial y} + 4\left(\frac{\partial u}{\partial x}\right)^2 + 2\frac{\partial v}{\partial x}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)\right] \\ &+\frac{\partial}{\partial y}\Bigg[\left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right)\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) + 2\frac{\partial u}{\partial x}\frac{\partial u}{\partial y} \\ &+2\frac{\partial v}{\partial x}\frac{\partial v}{\partial y}\Bigg]\Bigg\} + \frac{\alpha_2}{\rho}\frac{\partial}{\partial x}\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2\right] \end{aligned} \tag{6}$$

$$\begin{aligned} &\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + \frac{1}{\rho}\frac{\partial p}{\partial y} = \nu\nabla^2 v + \frac{\alpha_1}{\rho}\Bigg\{\frac{\partial}{\partial t}\left(\nabla^2 v\right) \\ &+\frac{\partial}{\partial x}\left[\left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right)\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) + 2\frac{\partial u}{\partial x}\frac{\partial u}{\partial y} + 2\frac{\partial v}{\partial x}\frac{\partial v}{\partial y}\right] \\ &+\frac{\partial}{\partial y}\Bigg[2u\frac{\partial^2 v}{\partial x\partial y} + 4\left(\frac{\partial v}{\partial y}\right)^2 + 2\frac{\partial u}{\partial y}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) \\ &+2v\frac{\partial^2 v}{\partial y^2}\Bigg]\Bigg\} + \frac{\alpha_2}{\rho}\frac{\partial}{\partial y}\left[\left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2\right] \end{aligned} \tag{7}$$

where $\nu = \dfrac{\mu}{\rho}$ is the kinematic viscosity.

Continuity equation (5) implies the existence of a streamfunction $\psi(x, y, t)$ such that

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \tag{8}$$

Substitution of (8) in equations (6) and (7) and elimination of pressure from the resulting equations using $p_{xy} = p_{yx}$ yields

$$\frac{\partial}{\partial t}\left(\nabla^2 \psi\right) - \frac{\alpha_1}{\rho}\frac{\partial}{\partial t}\left(\nabla^4 \psi\right) - \frac{\partial\left(\psi, \nabla^2 \psi\right)}{\partial(x, y)} + \frac{\alpha_1}{\rho}\frac{\partial\left(\psi, \nabla^4 \psi\right)}{\partial(x, y)} - \nu \nabla^4 \psi = 0 \tag{9}$$

Having obtained a solution of equation (9), the velocity components are given by (8) and the pressure can be found by integrating equations (6) and (7).

The shear stress component τ_{12} of the Cauchy stress T is given by

$$\begin{aligned}\tau_{12} = \mu\left[\frac{\partial^2 \psi}{\partial y^2} - \frac{\partial^2 \psi}{\partial x^2}\right] + \alpha_1 \Bigg[\frac{\partial \psi}{\partial y}\left(\frac{\partial^3 \psi}{\partial x \partial y^2} - \frac{\partial^3 \psi}{\partial x^3}\right) \\ - \frac{\partial \psi}{\partial x}\left(\frac{\partial^3 \psi}{\partial y^3} - \frac{\partial^3 \psi}{\partial x^2 \partial y}\right) + 2\frac{\partial^2 \psi}{\partial x \partial y}\frac{\partial^2 \psi}{\partial y^2} + 2\frac{\partial^2 \psi}{\partial x^2}\frac{\partial^2 \psi}{\partial x \partial y}\Bigg]\end{aligned} \tag{10}$$

3. Solutions in the Fixed Frame of Reference

Following Takemitsu and Matunobu [16], we assume that

$$\psi = k\left[x\, f(y) + g(y, t)\right] \tag{11}$$

We assume that the infinite plate at $y = 0$ is oscillating with velocity $U\cos\Omega t$ and that the fluid occupies the entire upper half plane $y > 0$. Furthermore, we assume the streamfunction far from the wall is given by $\psi = \frac{1}{2}\gamma y^2 + xy$ (see Stewart [2]). Thus, the boundary conditions are given by

$$f(0) = f'(0) = 0, \quad g(0, t) = 0, \quad g_y(0, t) = \frac{U}{k} e^{i\Omega t} \tag{12a}$$

$$f'(\infty) = 1, \quad g_y(\infty, t) = \gamma y \tag{12b}$$

where γ is a non-dimensional constant characterizing the obliqueness of oncoming flow. It is assumed that only the real part of a complex quantity has its physical meaning.

Substitution of equation (11) in (9) yields

$$\nu f^{(iv)} + k\left(f f''' - f' f''\right) - \frac{\alpha_1 k}{\rho}\left(f f^{(v)} - f' f^{(iv)}\right) = 0 \tag{13}$$

and

$$\begin{aligned}\nu \frac{\partial^4 g}{\partial y^4} - \frac{\partial^3 g}{\partial t \partial y^2} + \frac{\alpha_1}{\rho}\frac{\partial^5 g}{\partial t \partial y^4} + k\left(f \frac{\partial^3 g}{\partial y^3} - f'' \frac{\partial g}{\partial y}\right) \\ - \frac{\alpha_{1k}}{\rho}\left(f \frac{\partial^5 g}{\partial y^5} - f^{(iv)} \frac{\partial g}{\partial y}\right) = 0\end{aligned} \tag{14}$$

Integrating equations (13) and (14) once with respect to y using the conditions at infinity, we have

$$\nu f''' + k\left(f f'' - f'^2\right) - \frac{\alpha_1 k}{\rho}\left(f f^{(iv)} - 2 f' f''' + f''^2\right) = -k \tag{15}$$

and

$$\begin{aligned}\nu \frac{\partial^3 g}{\partial y^3} - \frac{\partial^2 g}{\partial t \partial y} + \frac{\alpha_1}{\rho}\frac{\partial^4 g}{\partial t \partial y^3} + k\left(f \frac{\partial^2 g}{\partial y^2} - f' \frac{\partial g}{\partial y}\right) \\ - \frac{\alpha_{1k}}{\rho}\left(f \frac{\partial^4 g}{\partial y^4} - f' \frac{\partial^3 g}{\partial y^3} + f'' \frac{\partial^2 g}{\partial y^2} - f''' \frac{\partial g}{\partial y}\right) \\ = 0\end{aligned} \tag{16}$$

Using the non-dimensional variables

$$\begin{aligned}&\eta = \sqrt{\frac{k}{\nu}}\, y, \quad \tau = \Omega t, \quad f(y) = \sqrt{\frac{\nu}{k}}\, F(\eta), \\ &g(y, t) = \frac{\nu}{k} G(\eta, \tau), \quad \varepsilon = \frac{U}{\sqrt{\nu k}}, \quad \beta = \frac{\Omega}{k}\end{aligned} \tag{17}$$

in equations (15) and (16), and boundary conditions (12a) and (12b), we obtain

$$F''' + F F'' - F'^2 - W_e\left(F F^{(iv)} - 2 F' F''' + F''^2\right) = -1 \tag{18a}$$

$$F(0) = 0, \quad F'(0) = 0, \quad F'(\infty) = 1 \tag{18b}$$

and

$$\frac{\partial^3 G}{\partial \eta^3}+F\frac{\partial^2 G}{\partial \eta^2}-F'\frac{\partial G}{\partial \eta}-We\left(F\frac{\partial^4 G}{\partial \eta^4}-F'\frac{\partial^3 G}{\partial \eta^3}\right.$$
$$\left.F''\frac{\partial^2 G}{\partial \eta^2}-F'''\frac{\partial G}{\partial \eta}\right)-\beta\frac{\partial^2 G}{\partial \tau\partial \eta}+We\beta\frac{\partial^4 G}{\partial \tau \partial \eta^3}=0 \qquad (19a)$$

$$G(0,\tau)=0,\quad G_\eta(0,\tau)=\varepsilon e^{i\tau},\quad G_{\eta\eta}(\infty,\tau)=\gamma \qquad (19b)$$

where $We=\frac{\alpha_1 k}{\rho\nu}$ is the Weissenberg number.

System (18 a-b) has been solved numerically by Garg and Rajagopal [23] and Ariel [24, 25]. Following Bellman and Kalaba [26] and Garg and Rajagopal [23], the quasi-linearized form of equation (18a) is

$$\begin{aligned}F_{n+1}^{(iv)}&=\frac{F_{n+1}'''}{F_n}\left(2F_n'+\frac{1}{We}\right)+F_{n+1}''\left(\frac{1}{We}-\frac{2F_n''}{F_n}\right)\\&+\frac{2F_{n+1}'}{F_n}\left(F'''-\frac{F_n'}{We}\right)\\&+\frac{F_{n+1}}{F_n^2}\left[F_n''^2-2F_n'F_n'''+\frac{F_n'^2-F_n'''-1}{We}\right]+\frac{F_n'''+2}{We\,F_n}\end{aligned} \qquad (20)$$

where the subscript n and $(n+1)$ represents the n^{th} and $(n+1)^{th}$ approximation to the solution. Since the above equation is non-homogeneous, the solution at any approximation level can be written as $= F_{\text{homogeneous}}+F_{\text{particular}}$. Further, the homogeneous solution, $F_{\text{homogeneous}}$, is a linear combination of two linearly independent solutions – namely F_{h_1} and F_{h_2} . The details of this technique are well described by Garg and Rajagopal [23].

Using the quasi-linearization technique described by Garg and Rajagopal [23], we find that $F''(0)=1.23259$ when $We=0$. This value is in good agreement with the value obtained by Takemitsu and Matunobu [16]. Numerical values of $F''(0)$ for different values of We are shown in Table 1. These values are in good agreement with the values obtained by Garg and Rajagopal [23] and Ariel [24]. Figure 1 shows the profiles of F' for various values of We. We observed that as the elasticity of the fluid increases, the velocity near the wall decreases.

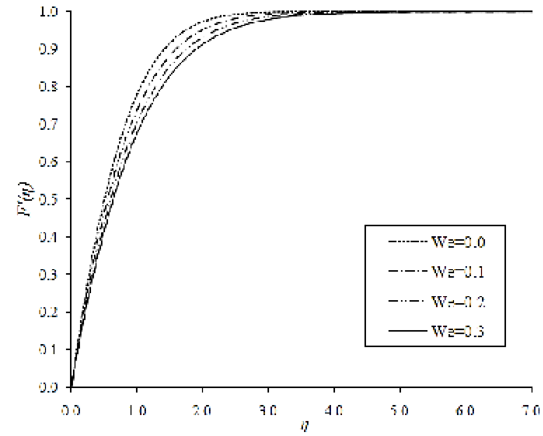

Figure 1. Variations of $F'(\eta)$ for various values of We.

Letting $G(\eta,\tau)=G_0(\eta)+\varepsilon G_1(\eta)e^{i\tau}$, then system (19) gives

$$\begin{aligned}&G_0'''+FG_0''-F'G_0'\\&\qquad -We\big(FG_0^{(iv)}-F'G_0'''+F''G_0''\\&\qquad\qquad -F'''G_0'\big)=0\end{aligned} \qquad (21a)$$

$$G_0(0)=0,\quad G_0'(0)=0,\quad G_0''(\infty)=\gamma \qquad (21b)$$

and

$$\begin{aligned}&G_1'''+FG_1''-F'G_1'-We\Big(FG_1^{(iv)}-F'G_1'''+F''G_1''\\&\qquad -F'''G_1'\big)-i\beta(G_1'-We\,G_1''')=0\end{aligned} \qquad (22a)$$

$$G_1(0)=0,\quad G_1'(0)=1,\quad G_1'(\infty)=0 \qquad (22b)$$

Letting $G_0'(\eta)=\gamma H_0(\eta)$, then system (21 a-b) gives

$$\begin{aligned}&H_0''+FH_0'-F'H_0-We(FH_0'''-F'H_0''+F''H_0'\\&\qquad -F'''H_0)=0\end{aligned} \qquad (23a)$$

$$H_0(0)=0,\quad H_0'(\infty)=1 \qquad (23b)$$

System (23 a-b) is solved numerically using a shooting method and it is found that for $We=0$, $H_0'(0)=0.607965$. Since $G_0''(0)=\gamma H_0'(0)$, then for $We=0$, $G_0''(0)=0.607965\,\gamma$ which is in good agreement with the value obtained by Takemitsu and Matunobu [16]. Numerical values of $H_0'(0)$ for different values of We are shown in Table 1. Figure 2 depicts the profiles of H_0' for various values of We.

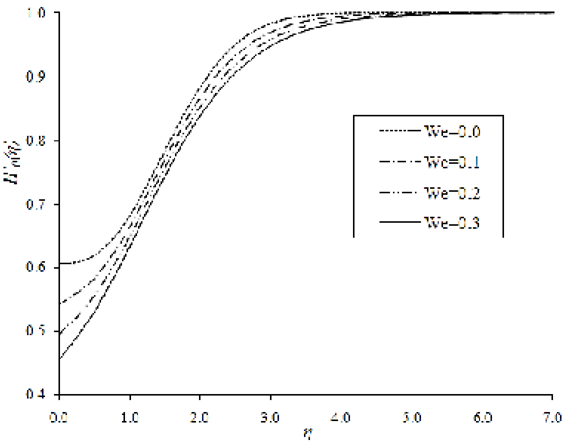

Figure 2. Variations of $H_0'(\eta)$ for various values of We.

Table 1. Numerical values of $F''(0)$, $H_0'(0)$, $\phi_0'(0)$, $\phi_1'(0)$ and $\phi_2'(0)$ for different values of We.

We	$F''(0)$	$H_0'(0)$	$\phi_0'(0)$	$\phi_1'(0)$	$\phi_2'(0)$
0.0	1.23259	0.60777	-0.81107	-0.49348	0.09471
0.1	1.13425	0.54392	-0.76774	0.50612	0.06023
0.2	1.05818	0.49546	-0.73291	0.51309	0.02785
0.3	0.99689	0.45677	-0.70364	0.51685	-0.00204
0.4	0.94588	0.42465	-0.67826	0.51881	-0.02953
0.5	0.90248	0.39774	-0.65619	0.51922	-0.05474
1.0	0.75276	0.30691	-0.57522	0.51155	-0.15428
2.0	0.59677	0.21662	-0.48170	0.48461	-0.27270
5.0	0.41288	0.12046	-0.35721	0.41638	-0.39192
8.0	0.33533	0.08503	-0.29916	0.37114	-0.40062
10	0.30283	0.07127	-0.27371	0.34885	-0.38807
20	0.21857	0.03978	-0.20475	0.28684	-0.37758
50	0.14008	0.01735	-0.13476	0.19505	0.16073
100	0.09951	0.00897	-0.09591	0.14291	0.33255
200	0.07053	0.00453	-0.06490	0.08305	1.48595
500	0.04469	0.00180	-0.02550	-0.04243	3.12615

Letting $\phi(\eta) = G_1'(\eta)$, then system (22 a-b) becomes

$$\phi'' + F\phi' - F'\phi - We(F\phi''' - F'\phi'' + F''\phi' - F'''\phi) - i\beta(\phi - We\,\phi'') = 0 \tag{24a}$$

$$\phi(0) = 1, \quad \phi(\infty) = 0 \tag{24b}$$

The only parameter in equation (24a) is the frequency β. Two series solutions valid for small and large β respectively will be obtained. For small values of the frequency β, we assume that

$$\phi(\eta) = \sum_{n=0}^{\infty} \beta^n \phi_n(\eta) = \phi_0(\eta) + i\beta\,\phi_1(\eta) + (i\beta)^2 \phi_2(\eta) + \cdots \tag{25}$$

where the numerical values for $\phi_0'(0)$, $\phi_1'(0)$ and $\phi_2'(0)$ are given in Table 1 for different values of We.

For large values of the frequency β, we let

$Y = \alpha\eta$, $\alpha = \sqrt{i\beta}$ and

$$\phi(\eta) = \sum_{n=0}^{\infty} \alpha^n \phi_n(\eta) = \phi_0(\eta) + \alpha\,\phi_1(\eta) + \alpha^2 \phi_2(\eta) + \cdots \tag{26}$$

and it was found that

$$\begin{aligned}\phi'(0) &= \phi_0'(0) + \alpha\phi_1'(0) + \alpha^2\phi_2'(0) + \alpha^3\phi_3'(0) + \cdots \\ &= -\frac{1}{\sqrt{1+m}} - \frac{(3-4m)}{8(1+m)}F''(0)\,\alpha^3 + \frac{3+4m}{16\sqrt{1+m}}\alpha^4 \\ &\quad - \frac{(40m^3 - 50m^2 + 28m - 33)F''^2(0)}{128(1+m)\sqrt{1+m}}\,\alpha^6 + \cdots\end{aligned} \tag{27}$$

where $m \neq 1$ and the numerical values of $F''(0)$ are given in Table 1 for different values of We. Figures 3-5 depict the variations of $\phi_0(\eta)$, $\phi_1(\eta)$ and $\phi_2(\eta)$ for various values of We.

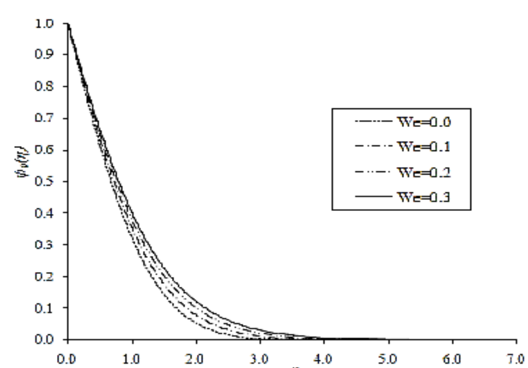

Figure 3. Variations of $\phi_0(\eta)$ for various values of We.

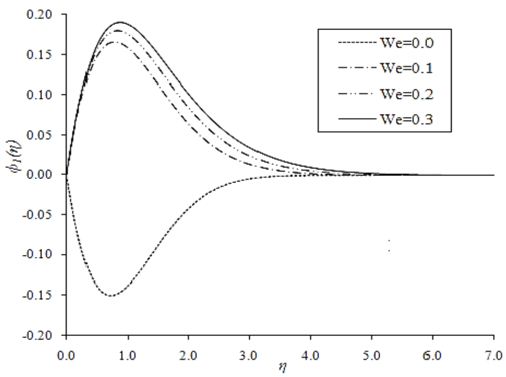

Figure 4. Variations of $\phi_1(\eta)$ for various values of We.

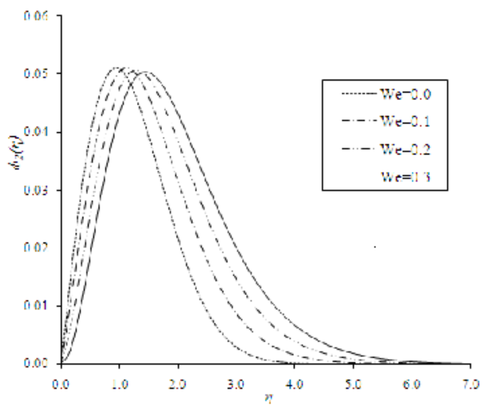

Figure 5. Variations of $\phi_2(\eta)$ for various values of We.

4. Solutions in the Moving Frame of Reference

We assume that the Cartesian coordinates (x, y) are moving with the plate, the x-axis is along the plate and the y-axis is normal to the plate. In this case, following Takemitsu and Matunobu [16], we assume that the streamfunction is given by

$$\psi = k\left[x\, f(y) + h(y,t)\right] \tag{28}$$

and the boundary conditions are given by

$$f(0) = f'(0) = 0, \qquad h(0,t) = 0, \qquad h_y(0,t) = 0 \tag{29a}$$

$$f'(\infty) = 1, \qquad h_y(\infty,t) = \gamma y - \frac{U}{k} e^{i\Omega t} \tag{29b}$$

We note that the flow is oscillating with velocity $-U\cos\Omega t$ at infinity. Using equation (28) in (9), equating different powers of x to zero and integrating once with respect to y using the conditions at infinity, we obtain

$$\nu f''' + k\left(f\, f'' - f'^2\right) - \frac{\alpha_1 k}{\rho}\left(f\, f^{(iv)} - 2 f' f''' + f''^2\right) = -k \tag{30}$$

and

$$\begin{aligned} &\nu \frac{\partial^3 h}{\partial y^3} - \frac{\partial^2 h}{\partial t \partial y} + \frac{\alpha_1}{\rho}\frac{\partial^4 h}{\partial t \partial y^3} + k\left(f\frac{\partial^2 h}{\partial y^2} - f'\frac{\partial h}{\partial y}\right) \\ &-\frac{\alpha_{1k}}{\rho}\left(f\frac{\partial^4 h}{\partial y^4} - f'\frac{\partial^3 h}{\partial y^3} + f''\frac{\partial^2 h}{\partial y^2} - f'''\frac{\partial h}{\partial y}\right) \\ &= \left(1 + \frac{i\Omega}{k}\right) U\, e^{i\Omega t} \end{aligned} \tag{31}$$

Non-dimensionalizing using

$$\begin{aligned} &\eta = \sqrt{\frac{k}{\nu}}\; y, \qquad \tau = \Omega t, \qquad f(y) = \sqrt{\frac{\nu}{k}}\, F(\eta), \\ &h(y,t) = \frac{\nu}{k} G(\eta, \tau), \qquad \beta = \frac{\Omega}{k}, \qquad \varepsilon = \frac{U}{\sqrt{\nu k}} \end{aligned} \tag{32}$$

we obtain

$$F''' + F\,F'' - F'^2 - W_e\left(F\,F^{(iv)} - 2F'F''' + F''^2\right) = -1 \tag{33a}$$

$$F(0) = 0, \qquad F'(0) = 0, \qquad F'(\infty) = 1 \tag{33b}$$

and

$$\begin{aligned} &\frac{\partial^3 G}{\partial \eta^3} + F\frac{\partial^2 G}{\partial \eta^2} - F'\frac{\partial G}{\partial \eta} - We\left(F\frac{\partial^4 G}{\partial \eta^4} - F'\frac{\partial^3 G}{\partial \eta^3}\right. \\ &\left.F''\frac{\partial^2 G}{\partial \eta^2} - F'''\frac{\partial G}{\partial \eta}\right) - \beta\frac{\partial^2 G}{\partial \tau \partial \eta} + We\beta\frac{\partial^4 G}{\partial \tau \partial \eta^3} = (1 + i\beta)\varepsilon\, e^{i\tau} \end{aligned} \tag{34a}$$

$$G(0,\tau) = 0, \quad G_\eta(0,\tau) = \varepsilon e^{i\tau}, \quad G_\eta(\infty,\tau) = \gamma\eta - \varepsilon\, e^{i\tau} \tag{34b}$$

System (33 a-b) has been solved numerically in section 3. Letting $G(\eta,\tau) = G_0(\eta) - \varepsilon\, H(\eta) e^{i\tau}$, system (34 a-b) gives

$$\begin{aligned} G_0''' + FG_0'' - F'G_0' - We\left(FG_0^{(iv)} - F'G_0''' + F''G_0''\right. \\ \left.-F'''G_0'\right) = 0 \end{aligned} \tag{35a}$$

$$G_0(0) = 0, \quad G_0'(0) = 0, \quad G_0''(\infty) = \gamma \tag{35b}$$

and

$$H''' + FH'' - F'H' - We\big(FH^{(iv)} - F'H''' + F''H'' \quad (36a)$$
$$-F'''H'\big) - i\beta(H' - We\, H''') = -1 - i\beta$$
$$H(0) = 0, \quad H'(0) = 1, \quad H'(\infty) = 1 \quad (36b)$$

Numerical solutions of system (35 a-b) have been obtained in section 3. It can easily be shown that the function

$$H' = \frac{F' + i\beta G_1' - i\beta}{1 - i\beta} \quad (37)$$

is a solution of system (36 a-b) since it satisfies both the equation and the boundary conditions. In equation (37), the functions F' and G_1' have been found in section 3.

4. Discussion and Conclusions

The unsteady second grade stagnation-point flow impinging obliquely on an oscillatory flat plate is studied. Numerical results for this flow are found for various values of the Weissenberg number We. Figure 1 shows the variations of $F'(\eta)$ for various values of We. The effect of the Weissenberg number, We, is to decrease the velocity $F'(\eta)$ near the wall as it increases. Figure 2 depicts the variations of $H_0'(\eta)$ for various values of We and shows that $H_0'(\eta)$ decreases near the wall as We is increasing. The variations of $\phi_0(\eta)$ with various values of We are shown in Figure 3. From this figure we observed that $\phi_0(\eta)$ is decreasing as We is incresing. Figure 4 shows the variations of $\phi_1(\eta)$ for various values of We and Figure 5 depicts the variations of $\phi_2(\eta)$ for various values of We. From Table 1, $F''(0)$ is decreasing as We is increasing.

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